NCERT Class 10 Maths Solutions: Chapter 1 - Real Numbers

Ex 1.1 Question 1.

Express each number as product of its prime factors:
(i) 140
(ii) 156
(iii) 3825
(iv) 5005
(v) 7429

Answer.

(i) $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$
(ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$
(iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$
(iv) $5005 = 5 \times 7 \times 11 \times 13$
(v) $7429 = 17 \times 19 \times 23$

Ex 1.1 Question 2.

Find the LCM and HCF of the following pairs of integers and verify that $\mathrm{LCM} \times \mathrm{HCF} =$ product of the two numbers.
(i) 26 and 91
(ii) 510 and 92
(iii) 336 and 54

Answer.

(i) 26 and 91
$26 = 2 \times 13$
$91 = 7 \times 13$
$\mathrm{HCF}(26, 91) = 13$
$\mathrm{LCM}(26, 91) = 2 \times 7 \times 13 = 182$

Product of two numbers 26 and $91 = 26 \times 91 = 2366$
$HCF \times LCM = 13 \times 182 = 2366$

Hence, product of two numbers $= HCF \times LCM$

(ii) 510 and 92
$510 = 2 \times 3 \times 5 \times 17$
$92 = 2 \times 2 \times 23$
$\mathrm{HCF}(510, 92) = 2$
$\mathrm{LCM}(510, 92) = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of two numbers 510 and $92 = 510 \times 92 = 46920$
$HCF \times LCM = 2 \times 23460 = 46920$

Hence, product of two numbers $= HCF \times LCM$

(iii) 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

$\mathrm{HCF}(336, 54) = 2 \times 3 = 6$

$\mathrm{LCM}(336, 54) = 2^4 \times 3^3 \times 7 = 3024$

Product of two numbers 336 and 54 $= 336 \times 54 = 18144$

$HCF \times LCM = 6 \times 3024 = 18144$

Hence, product of two numbers = $HCF \times LCM$

Ex 1.1 Question 3.

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

(ii) 17,23 and 29

(iii) 8,9 and 25

Answer.

(i) 12,15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$\mathrm{HCF}(12, 15, 21) = 3$

$\mathrm{LCM}(12, 15, 21) = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17,23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

$\mathrm{HCF}(17, 23, 29) = 1$

$\mathrm{LCM}(17, 23, 29) = 17 \times 23 \times 29 = 11339$

(iii) 8,9 and 25

$8 = 2 \times 2 \times 2 = 2^3$

$9 = 3 \times 3 = 3^2$

$25 = 5 \times 5 = 5^2$

$\mathrm{HCF}(8, 9, 25) = 1$

$\mathrm{LCM}(8, 9, 25) = 2^3 \times 3^2 \times 5^2 = 1800$

Ex 1.1 Question 4.

Given that HCF $(306, 657) = 9$, find LCM $(306, 657)$.

Answer.

$\mathrm{HCF}(306, 657) = 9$

We know that, $\mathrm{LCM} \times \mathrm{HCF} =$ Product of two numbers

$$LCM \times HCF = 306 \times 657$$

$$LCM = \frac{306 \times 657}{HCF} = \frac{306 \times 657}{9}$$

$$\mathrm{LCM}(306, 657) = 22338$$

Ex 1.1 Question 5.

Check whether 6^n can end with the digit 0 for any natural number $\boldsymbol{n}$.

Answer.

If any number ends with the digit 0 , it should be divisible by 10 .

In other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n.

Hence, for any value of $n, 6^n$ will not be divisible by 5 .

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Ex 1.1 Question 6.

Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Answer.

Numbers are of two types - prime and composite.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13$
$= 13 \times (7 \times 11 + 1)$
$= 13 \times (77 + 1)$
$= 13 \times 78 = 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors other than 1 and number itself.
Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$
$= 5 \times (1008 + 1) = 5 \times 1009$

1009 cannot be factorized further
Therefore, the given expression has 5 and 1009 as its factors other than 1 and number itself.
Hence, it is a composite number.

Ex 1.1 Question 7.

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer.

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$18 = 2 \times 3 \times 3 \text{ And, } 12 = 2 \times 2 \times 3$

LCM of 12 and $18 = 2 \times 2 \times 3 \times 3 = 36$
Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

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Ex 1.2 Question 1.

Prove that $\sqrt{5}$ is irrational.

Answer.

Let us prove $\sqrt{5}$ irrational by contradiction.
Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers $\boldsymbol{a}$ and $\boldsymbol{b}$ ($b \neq 0$) such that $\sqrt{5}=\frac{a}{b}$
$\Rightarrow b\sqrt{5}=a$

Squaring both sides, we get
$\Rightarrow 5b^2=a^2$

It means that 5 is factor of a^2
Hence, 5 is also factor of a by Theorem.... (2)
If, $\mathbf{5}$ is factor of $\boldsymbol{a}$, it means that we can write $\boldsymbol{a}=5\boldsymbol{c}$ for some integer $\boldsymbol{c}$.
Substituting value of $\boldsymbol{a}$ in (1),
$5b^2=25c^2 \Rightarrow b^2=5c^2$

It means that 5 is factor of b^2.
Hence, 5 is also factor of $\boldsymbol{b}$ by Theorem. ... (3)
From (2) and (3), we can say that 5 is factor of both $\boldsymbol{a}$ and $\boldsymbol{b}$.
But, $\boldsymbol{a}$ and $\boldsymbol{b}$ are co-prime.
Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

Ex 1.2 Question2.

Prove that $(3+2\sqrt{5})$ is irrational.

Answer.

We will prove this by contradiction.
Let us suppose that $(3+2\sqrt{5})$ is rational.
It means that we have co-prime integers $\boldsymbol{a}$ and $\boldsymbol{b}(b \neq 0)$ such that

$$\frac{a}{b}=3+2\sqrt{5} \Rightarrow \frac{a}{b}-3=2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{b}=2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b}=\sqrt{5}\ldots(1)$$

$\boldsymbol{a}$ and $\boldsymbol{b}$ are integers.
It means L.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational. It is not possible. Therefore, our supposition is wrong. $(3+2\sqrt{5})$ cannot be rational.

Hence, $(3+2\sqrt{5})$ is irrational.

Ex 1.2 Question3.

Prove that the following are irrationals.

(i) $\frac{1}{\sqrt{2}}$

(ii) $7\sqrt{5}$

(iii) $6+\sqrt{2}$

Answer.

(i) We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers a and $b(b \neq 0)$ such that

$$\frac{1}{\sqrt{2}}=\frac{a}{b}$$

$$\Rightarrow \sqrt{2}=\frac{b}{a}\ldots$$

R.H.S of (1) is rational but we know that $\sqrt{2}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $\frac{1}{\sqrt{2}}$ cannot be rational.

Hence, it is irrational.

(ii) We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers a and $b(b \neq 0)$ such that

$$7\sqrt{5}=\frac{a}{b}$$

$$\Rightarrow \sqrt{5}=\frac{a}{7b}\ldots$$

R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

(iii) We will prove $6+\sqrt{2}$ irrational by contradiction.

Let us suppose that $(6+\sqrt{2})$ is rational.

It means that we have co-prime integers $\boldsymbol{a}$ and $\boldsymbol{b}(b \neq 0)$ such that

$$6+\sqrt{2}=\frac{a}{b}$$

$$\Rightarrow \sqrt{2}=\frac{a}{b}-6$$

$$\Rightarrow \sqrt{2}=\frac{a-6b}{b}\ldots$$

$\boldsymbol{a}$ and $\boldsymbol{b}$ are integers.

It means L.H.S of (1) is rational but we know that $\sqrt{2}$ is irrational. It is not possible. Therefore, our supposition is wrong. $(6+\sqrt{2})$ cannot be rational. Hence, $(6+\sqrt{2})$ is irrational.